Vinokulmainen kolmio

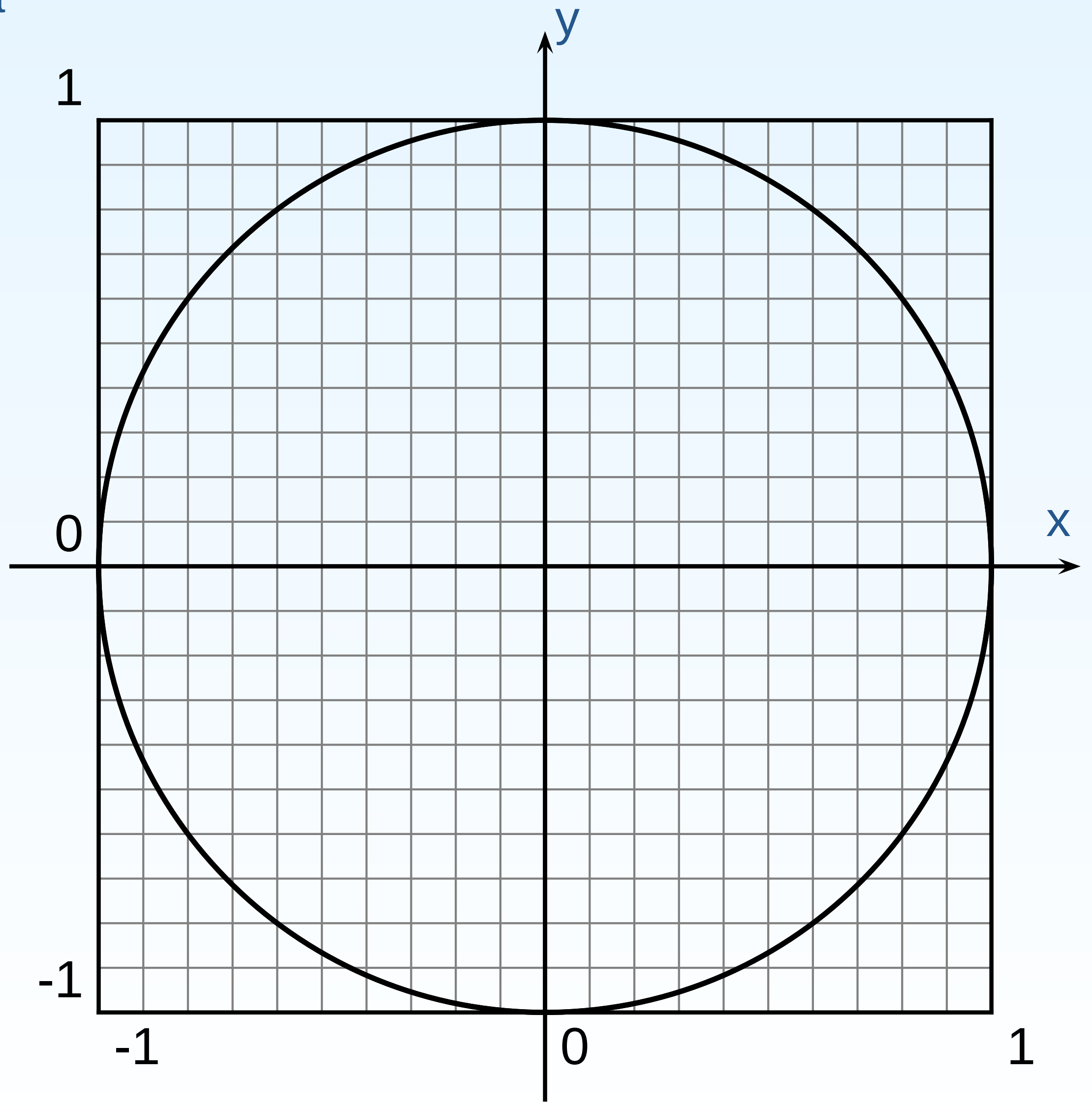
Hannu Lehto

Lahden Lyseon lukio

LAHDEN LYSEO

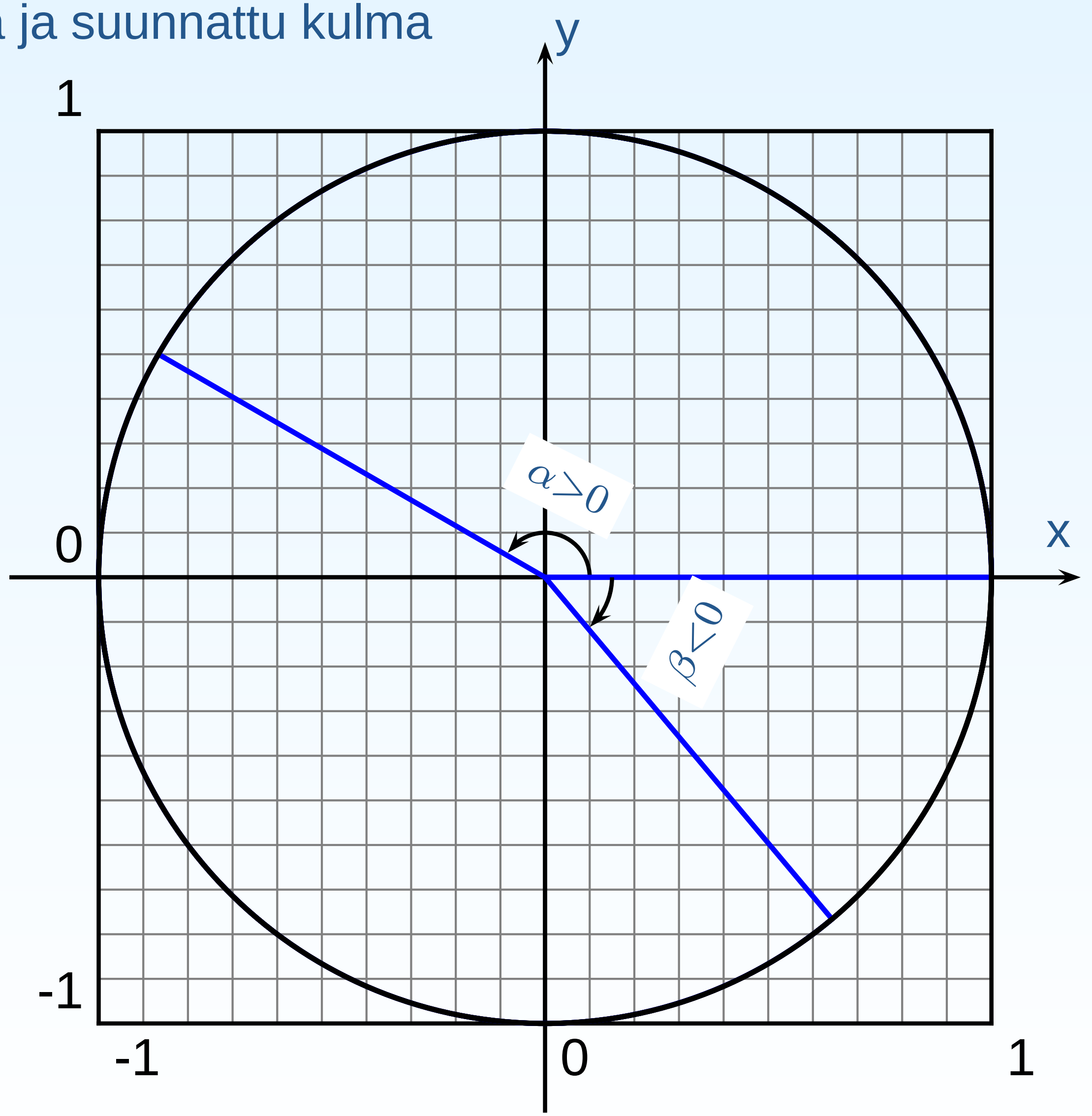
Yksikköympyrä ja suunnattu kulma

Yksikköympyrä

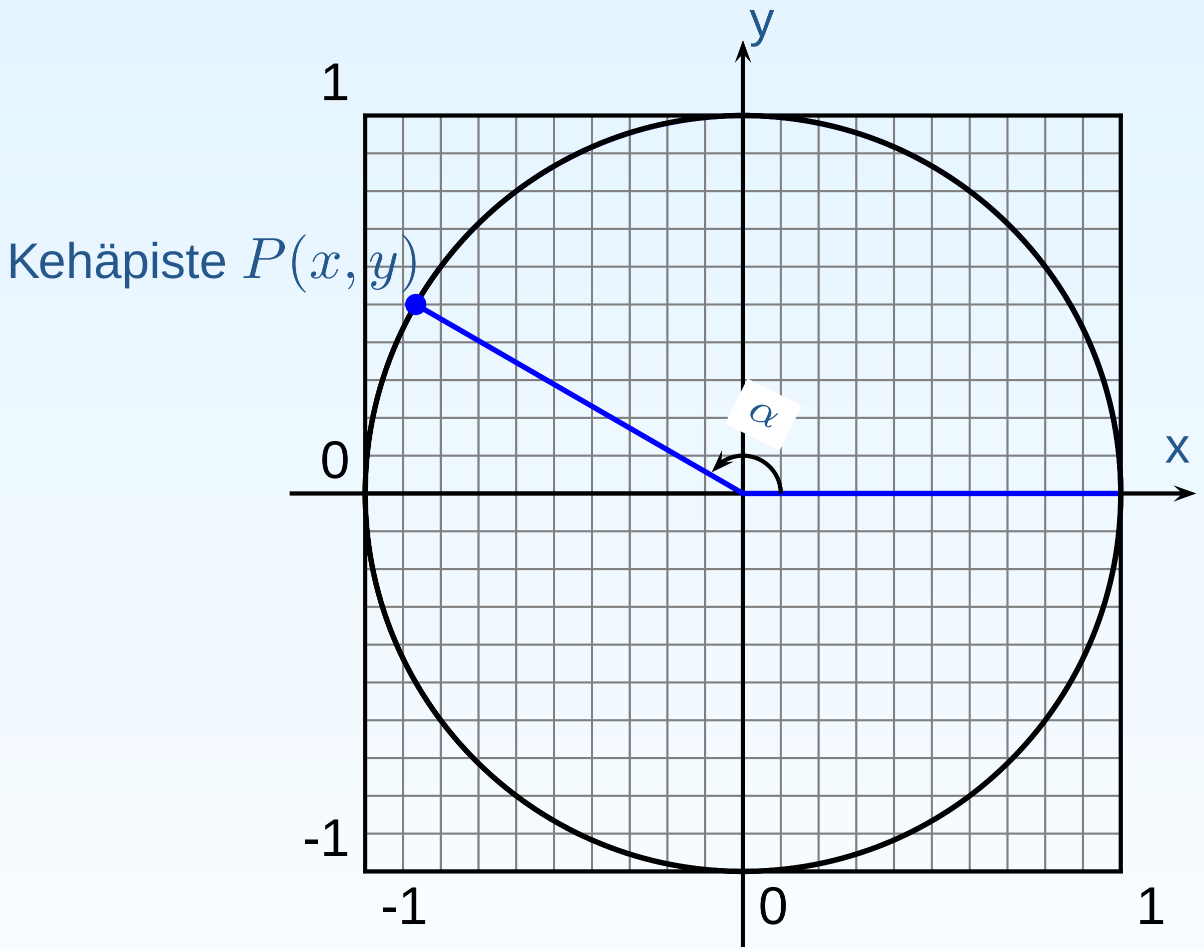


Yksikköympyrä ja suunnattu kulma

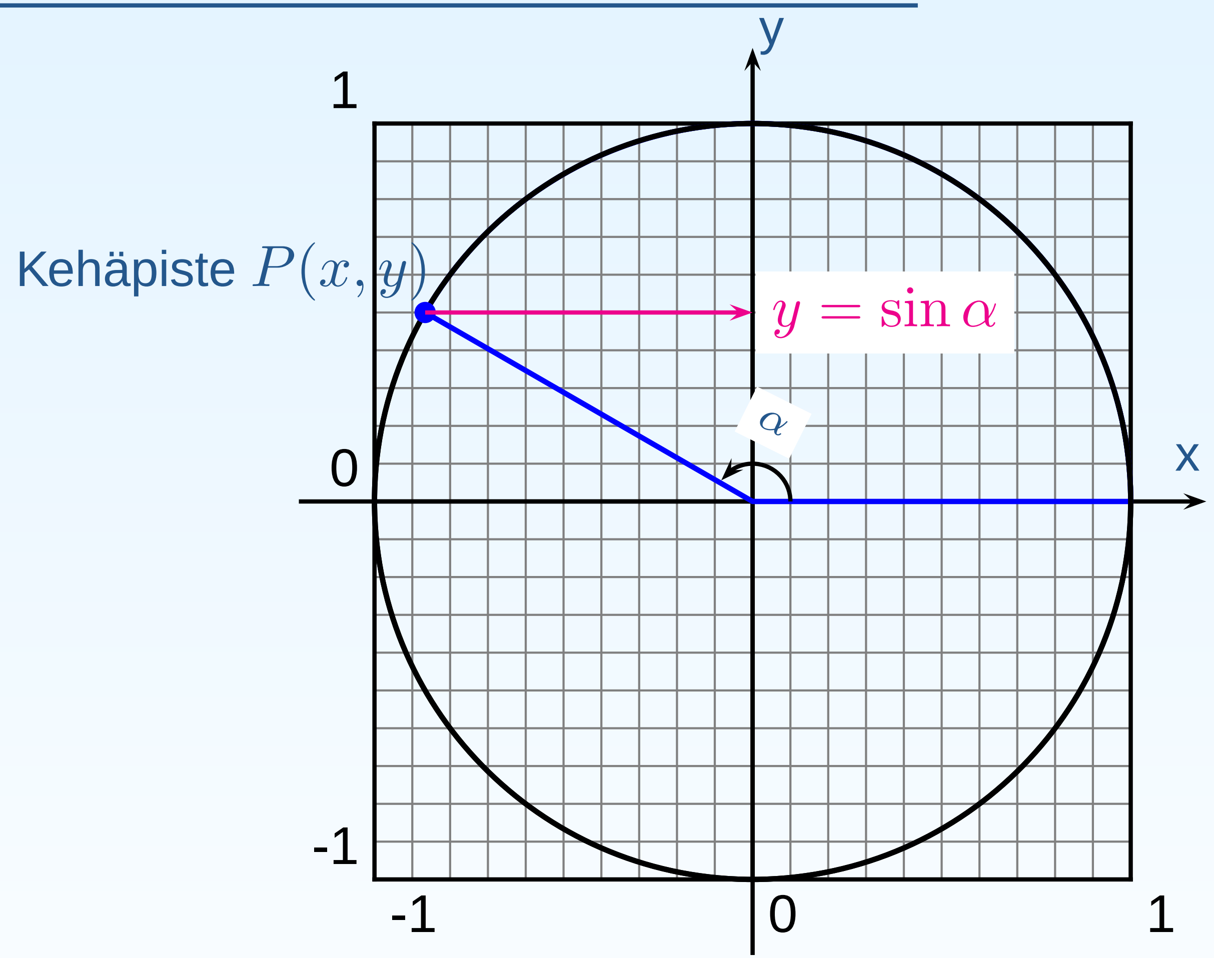
Yksikköympyrä ja suunnattu kulma



Trigonometristen funktioiden määritelmät



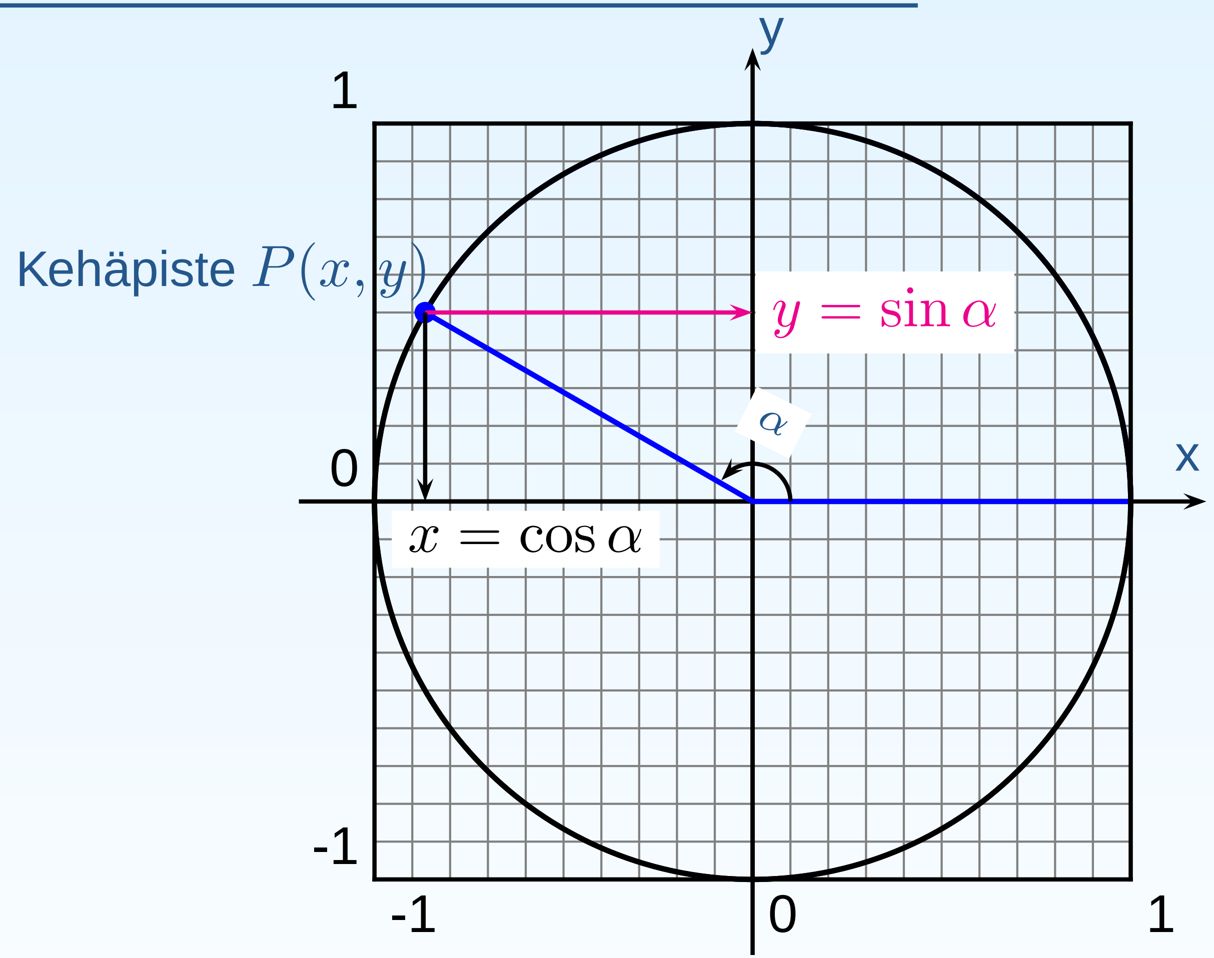
Trigonometristen funktioiden määritelmät



Määritelmä.

$\sin\alpha = y$ (kehäpisteen y-koordinaatti) Sovelma

Trigonometristen funktioiden määritelmät

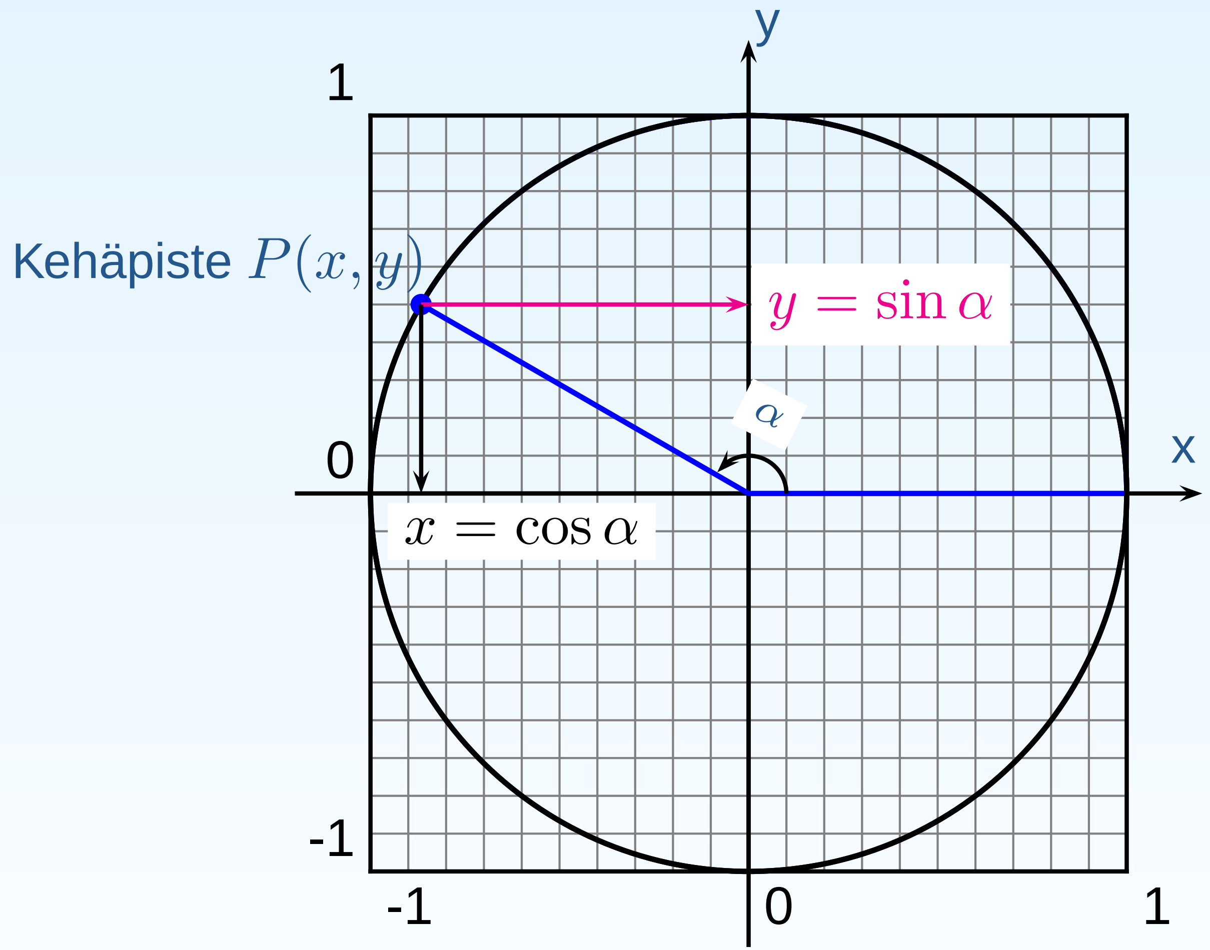


Määritelmä.

$\sin\alpha = y$ (kehäpisteen y-koordinaatti) Sovelma

$\cos\alpha = x$ (kehäpisteen x-koordinaatti) Sovelma

Trigonometristen funktioiden määritelmät



Määritelmä.

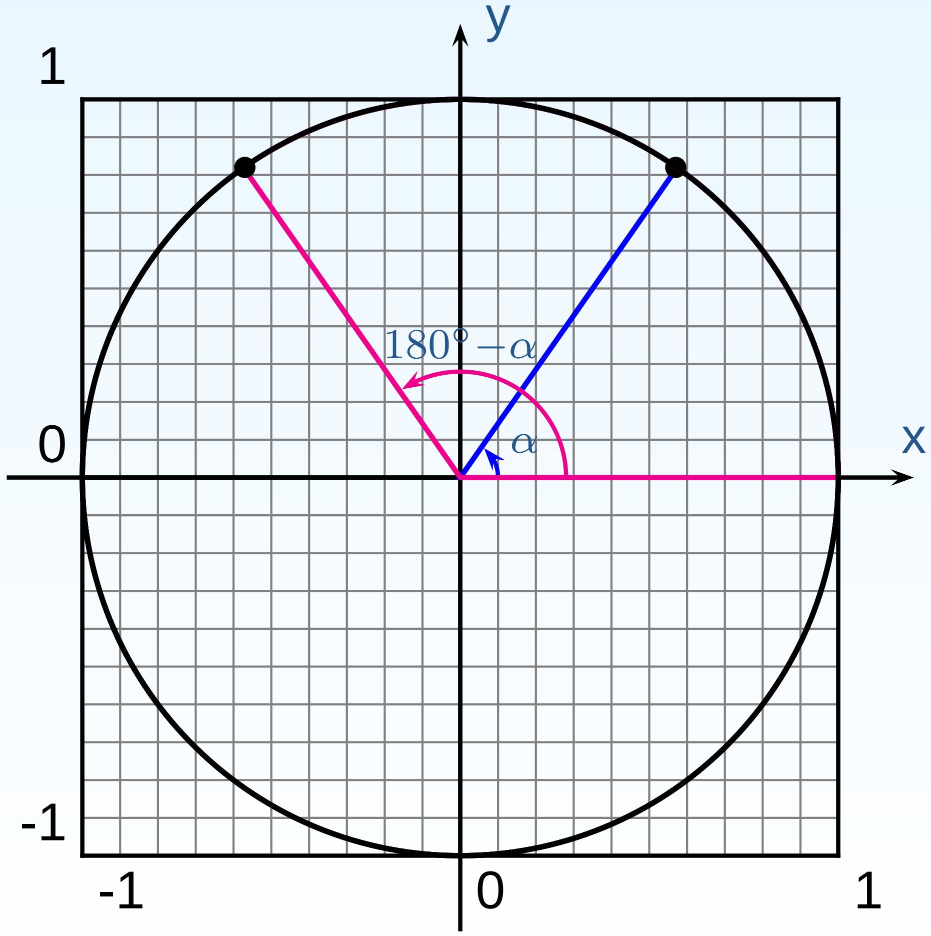
$\sin\alpha = y$ (kehäpisteen y-koordinaatti) Sovelma

$\cos\alpha = x$ (kehäpisteen x-koordinaatti) Sovelma

$$\tan\alpha = \frac{y}{x} = \frac{\sin\alpha}{\cos\alpha}, x \neq 0$$

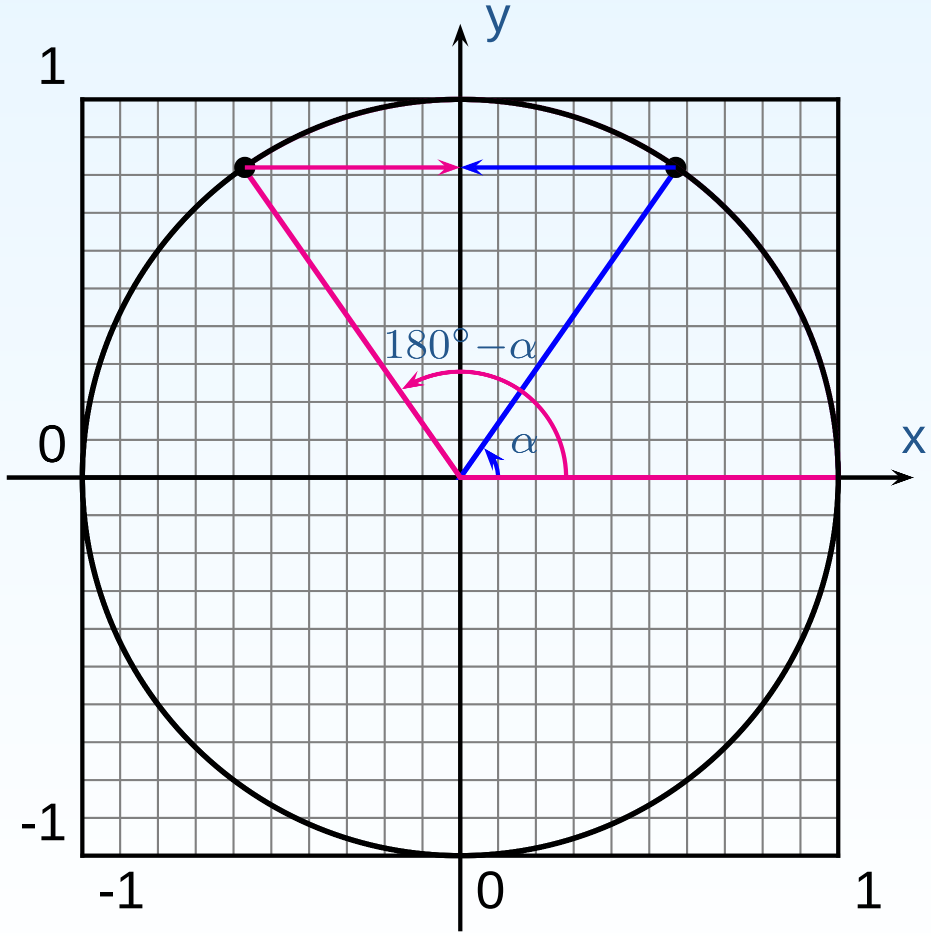
Suplementtikulman trigonometriset funktiot

- $\sin(180° - \alpha) =$
- $\cos(180° - \alpha) =$
- $\tan(180° - \alpha) =$



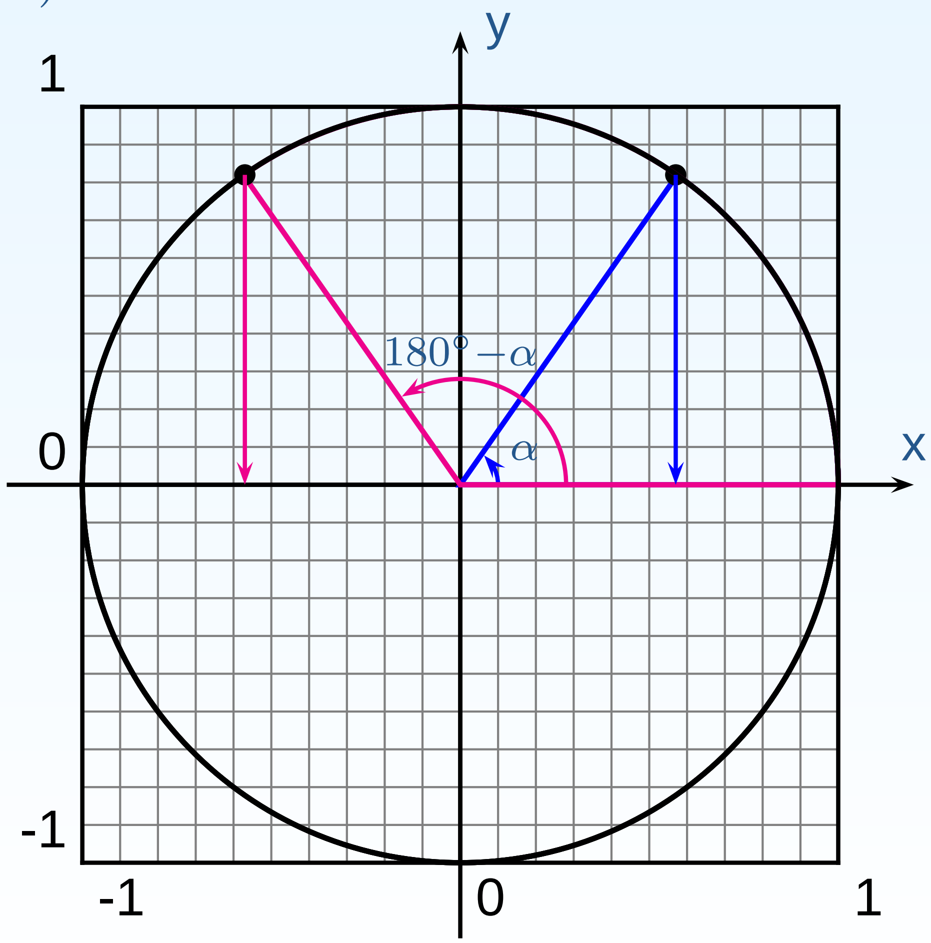
Suplementtikulman trigonometriset funktiot

- $\sin(180° - \alpha) = \sin\alpha$
- $\cos(180° - \alpha) =$
- $\tan(180° - \alpha) =$



Suplementtikulman trigonometriset funktiot

- $\sin(180° - \alpha) = \sin \alpha$
- $\cos(180° - \alpha) = -\cos \alpha$
- $\tan(180° - \alpha) =$

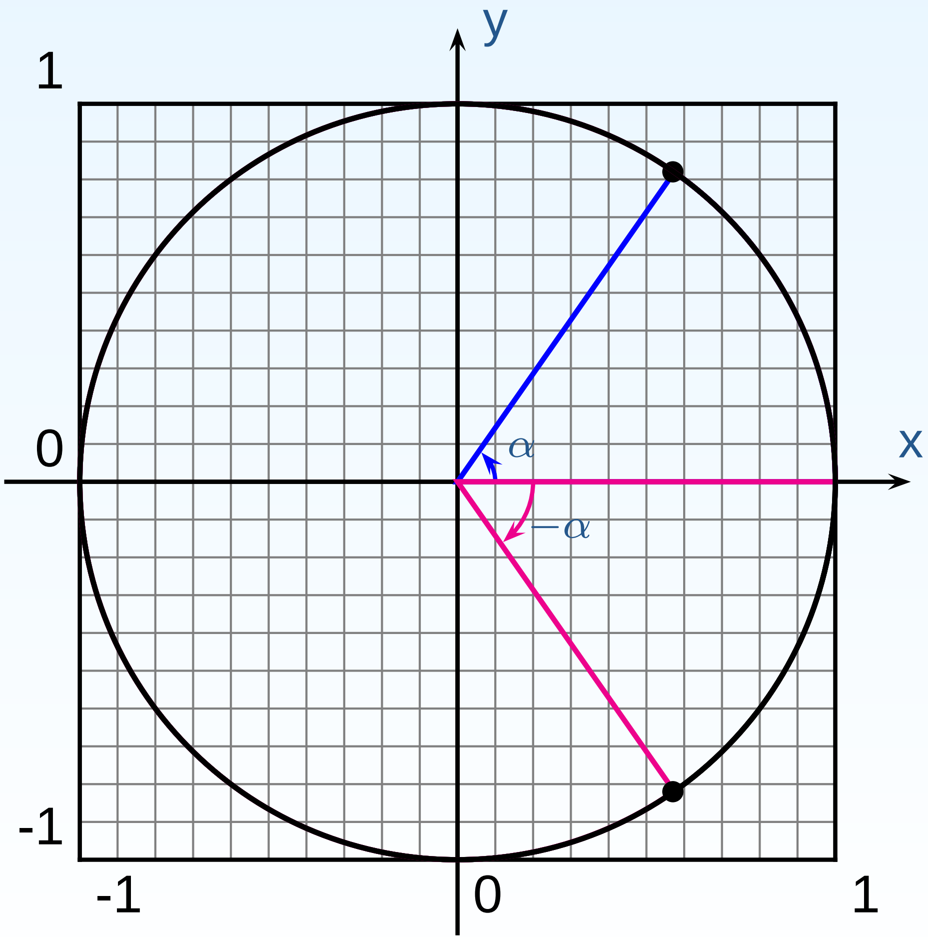


Suplementtikulman trigonometriset funktiot

- $\sin(180^\circ - \alpha) = \sin\alpha$
- $\cos(180^\circ - \alpha) = -\cos\alpha$
- $\tan(180^\circ - \alpha) = \dfrac{\sin(180^\circ - \alpha)}{\cos(180^\circ - \alpha)} = \dfrac{\sin\alpha}{-\cos\alpha} = -\tan\alpha$

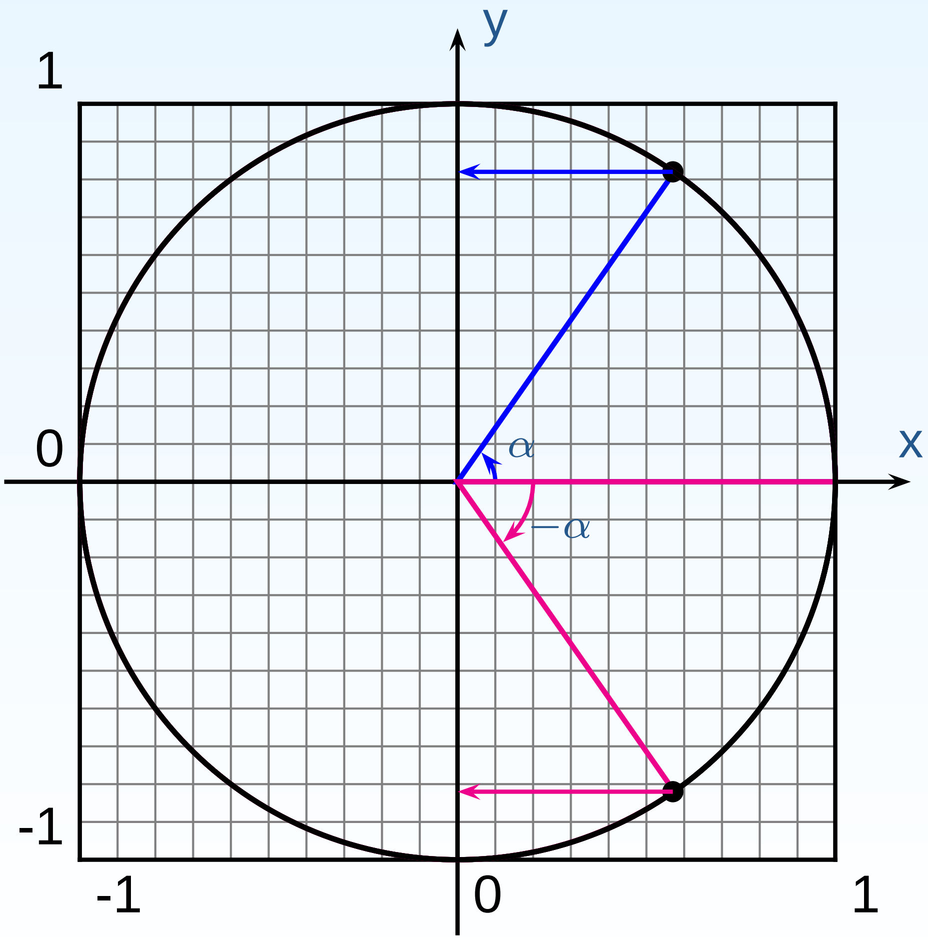
Vastakulmien trigonometriset funktiot

- $\sin(-\alpha) =$
- $\cos(-\alpha) =$
- $\tan(-\alpha) =$



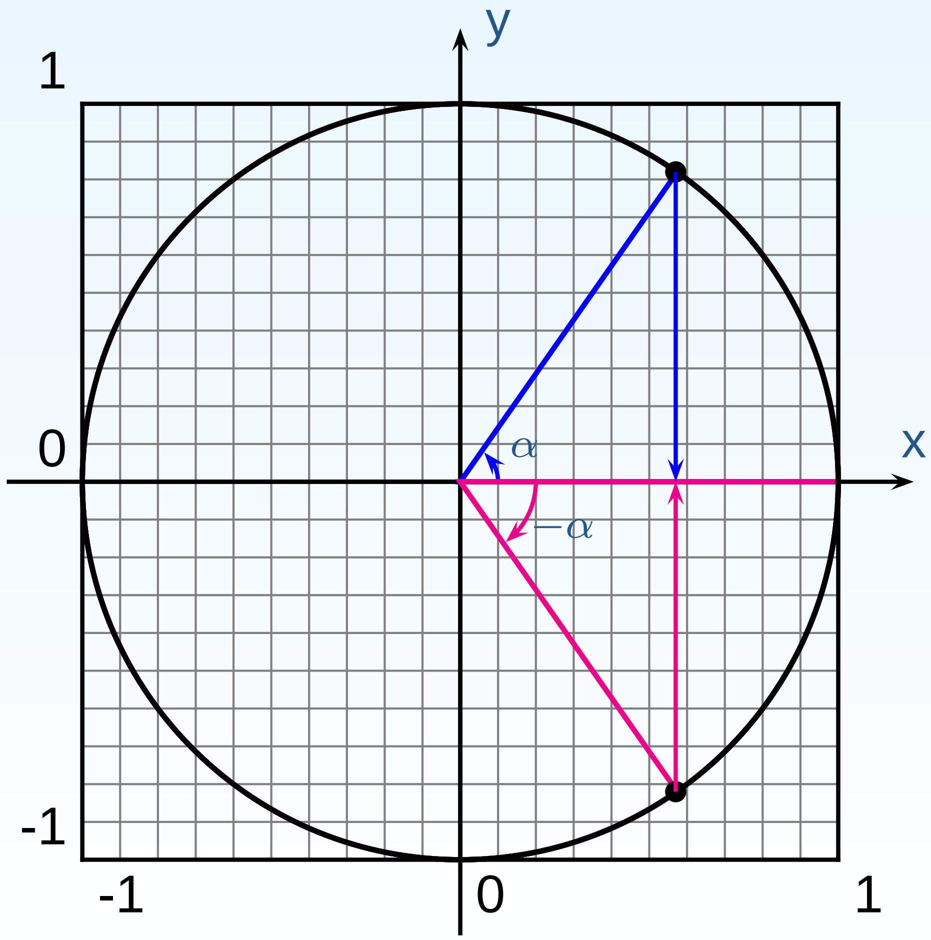
Vastakulmien trigonometriset funktiot

- $\sin(-\alpha) = -\sin\alpha$ (pariton funktio)
- $\cos(-\alpha) =$
- $\tan(-\alpha) =$



Vastakulmien trigonometriset funktiot

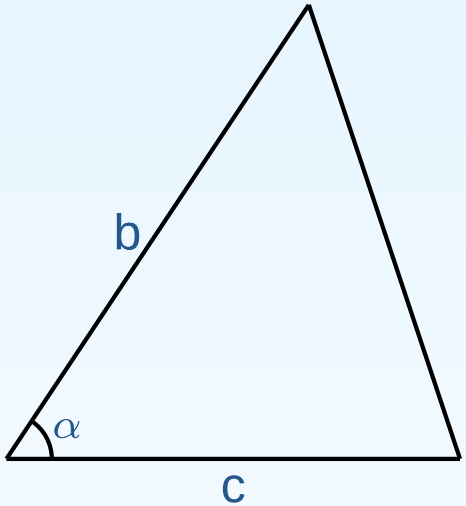
- $\sin(-\alpha) = -\sin\alpha$ (sini on pariton funktio)
- $\cos(-\alpha) = \cos\alpha$ (kosini on parillinen funktio)
- $\tan(-\alpha) =$



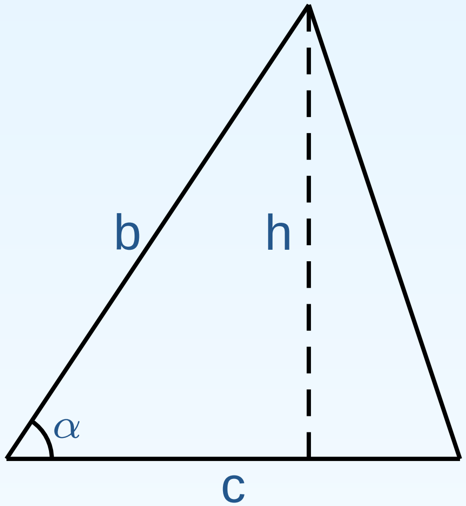
Vastakulmien trigonometriset funktiot

- $\sin(-\alpha) = -\sin\alpha$ (sini on pariton funktio)
- $\cos(-\alpha) = \cos\alpha$ (kosini on parillinen funktio)
- $\tan(-\alpha) = \dfrac{\sin(-\alpha)}{\cos(-\alpha)} = \dfrac{-\sin\alpha}{\cos\alpha} = -\tan\alpha$

Kolmion ala

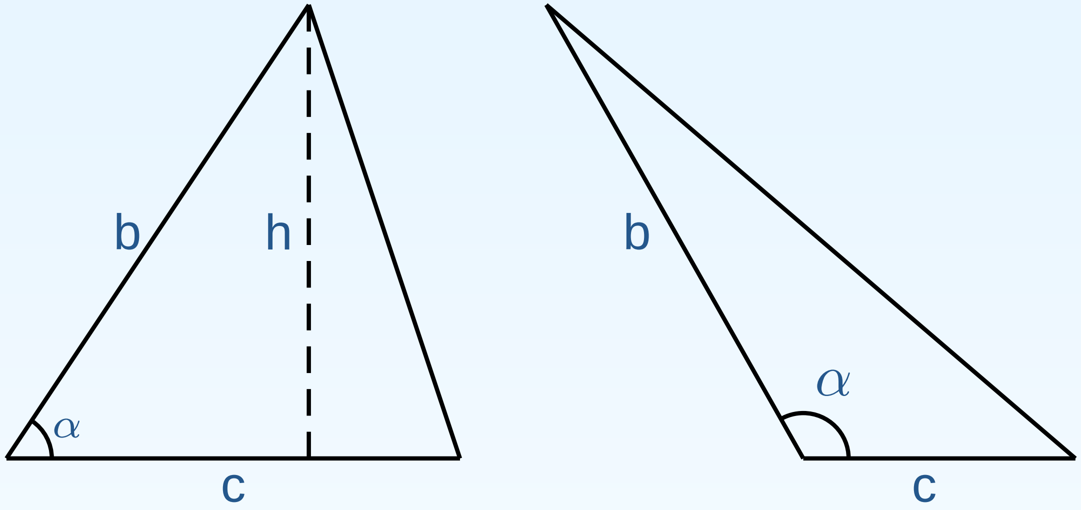


Kolmion ala



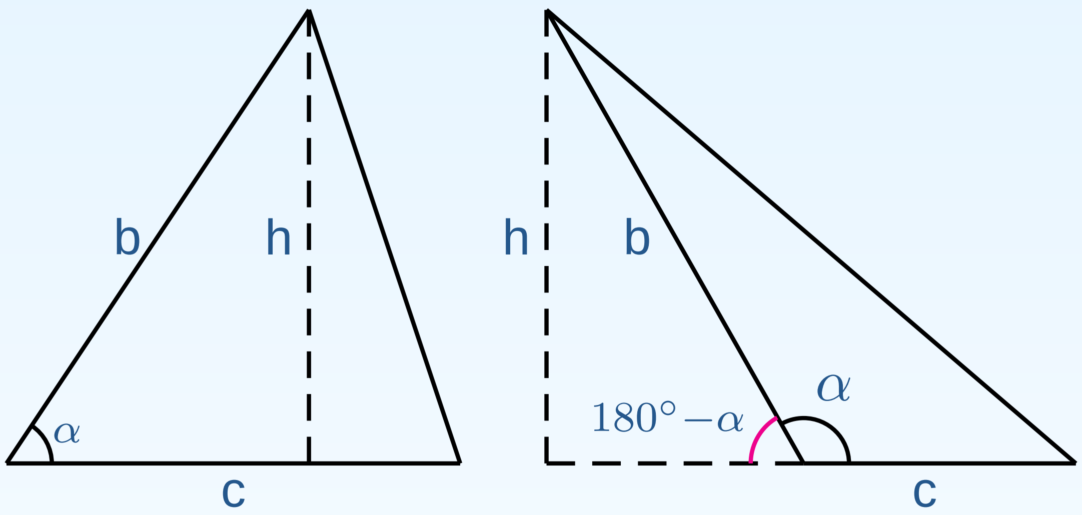
$$
\begin{aligned}
A &= \frac{1}{2}ch \\
&= \frac{1}{2}cb\sin\alpha
\end{aligned}
$$

Kolmion ala



$$A = \frac{1}{2}ch$$
$$= \frac{1}{2}cb\sin\alpha$$

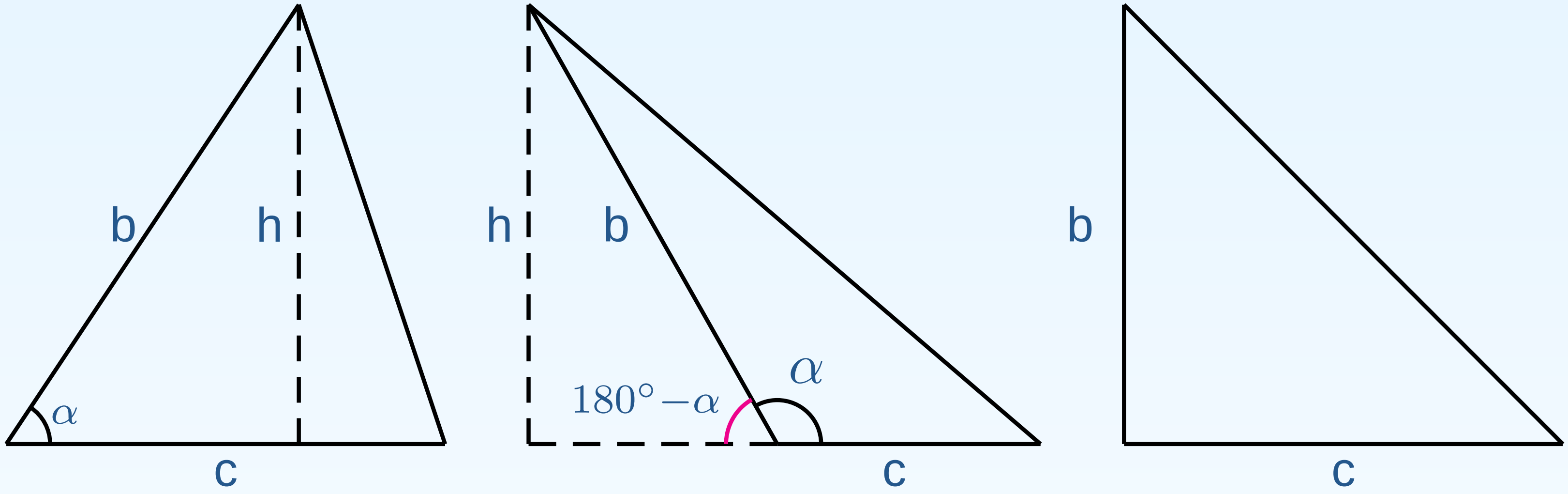
Kolmion ala



$$\begin{aligned} A &= \tfrac{1}{2}ch \\ &= \tfrac{1}{2}cb\sin\alpha \end{aligned}$$

$$\begin{aligned} A &= \tfrac{1}{2}ch \\ &= \tfrac{1}{2}cb\sin(180^\circ - \alpha) \\ &= \tfrac{1}{2}cb\sin\alpha \end{aligned}$$

Kolmion ala

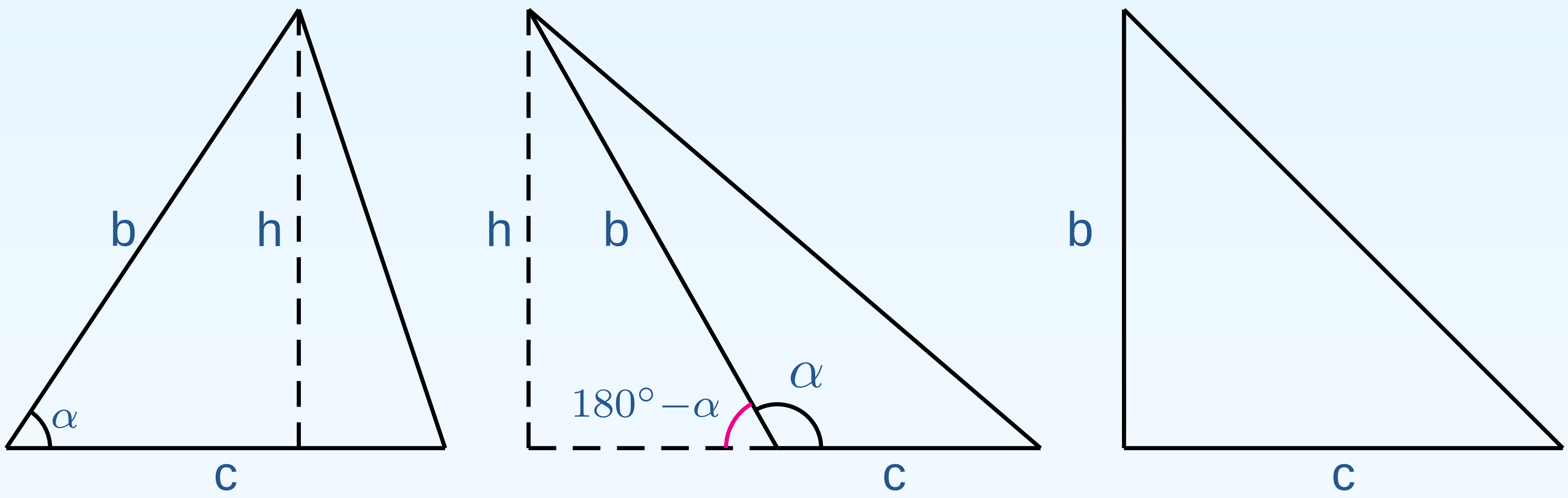


$$A = \frac{1}{2}ch = \frac{1}{2}cb\sin\alpha$$

$$A = \frac{1}{2}ch = \frac{1}{2}cb\sin(180^\circ - \alpha) = \frac{1}{2}cb\sin\alpha$$

$$A = \frac{1}{2}cb = \frac{1}{2}cb\sin 90^\circ$$

Kolmion ala



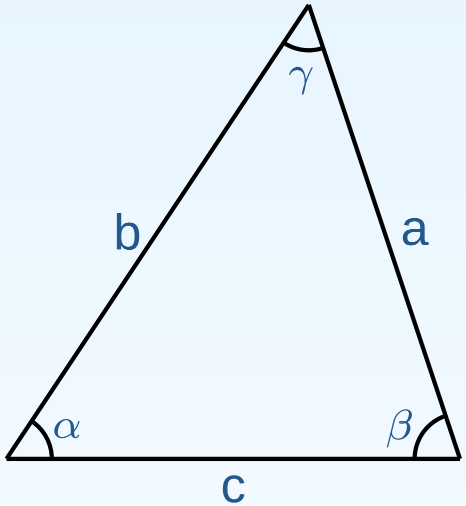
$$A = \frac{1}{2}ch = \frac{1}{2}cb\sin\alpha$$

$$A = \frac{1}{2}ch = \frac{1}{2}cb\sin(180^\circ - \alpha) = \frac{1}{2}cb\sin\alpha$$

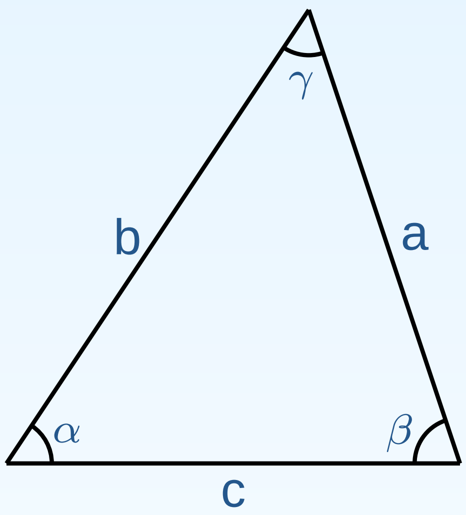
$$A = \frac{1}{2}cb = \frac{1}{2}cb\sin 90^\circ$$

Lause. $\boxed{A = \frac{1}{2}bc\sin\alpha}$

Sinilause

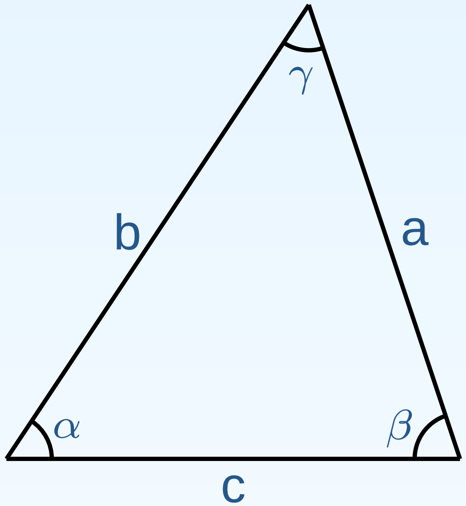


Sinilause



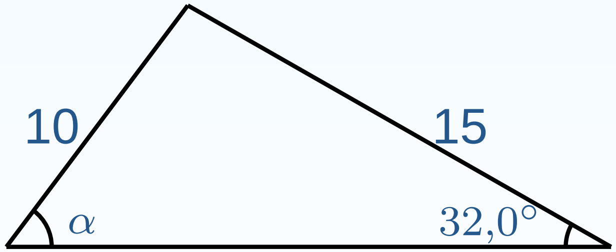
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Sinilause

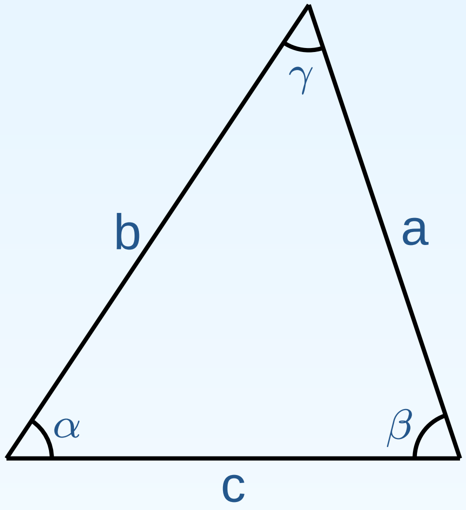


$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Esimerkki. Laske kulma α. Ilmoita vastaus $0,1^\circ$:een tarkkuudella.

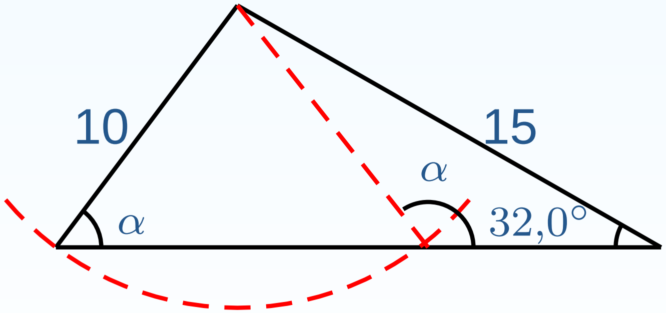


Sinilause



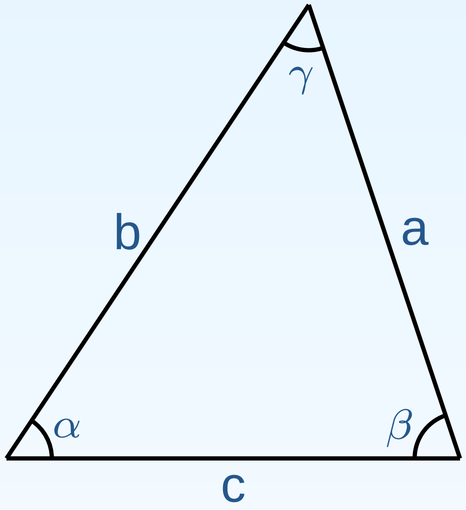
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Esimerkki. Laske kulma α. Ilmoita vastaus $0,1°$:een tarkkuudella.



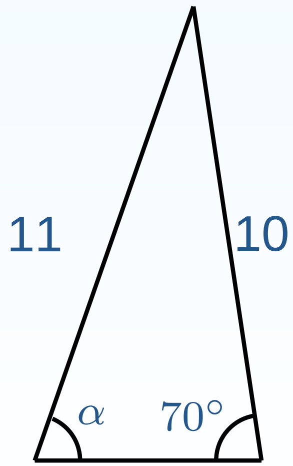
Vastaus. $\alpha = 52,6°$ tai $\alpha = 127,4°$

Sinilause

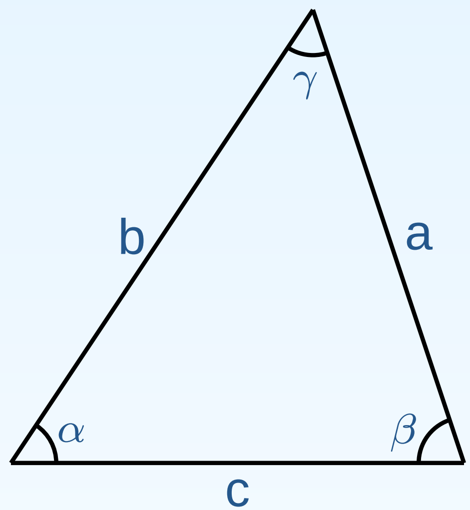


$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Esimerkki. Laske kulma α. Ilmoita vastaus $0,1^\circ$:een tarkkuudella.

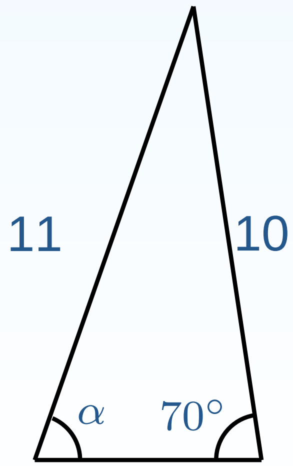


Sinilause



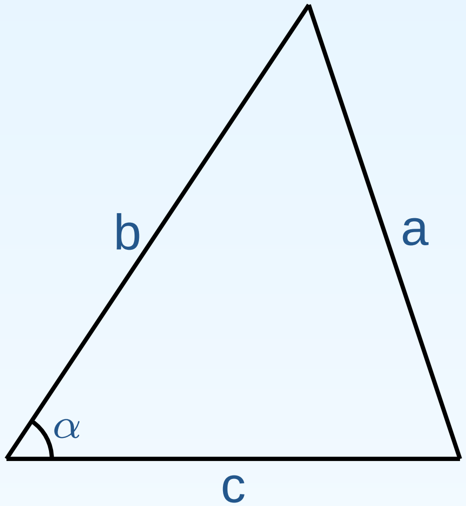
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Esimerkki. Laske kulma α. Ilmoita vastaus $0,1^\circ$:een tarkkuudella.

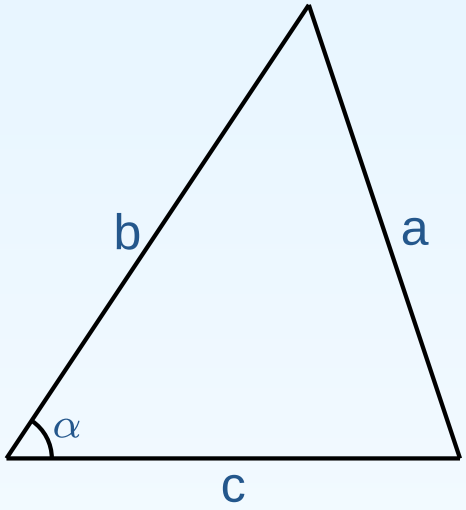


Vastaus. $\alpha = 58,7^\circ$.
Huomaa, että $180^\circ - 58,7^\circ = 121,3^\circ$ ei kelpaa ratkaisuksi, koska $121,3^\circ + 70^\circ > 180^\circ$!

Kosinilause eli laajennettu Pythagoraan lause

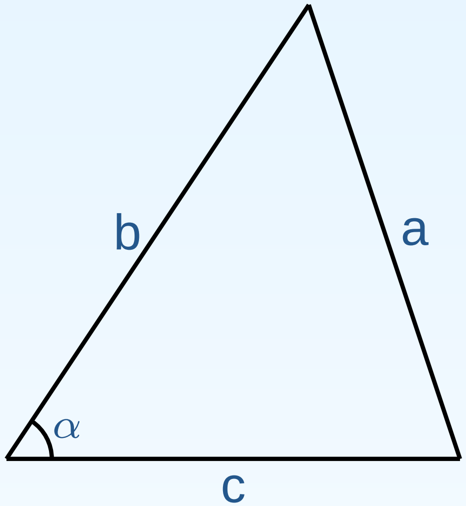


Kosinilause eli laajennettu Pythagoraan lause



$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

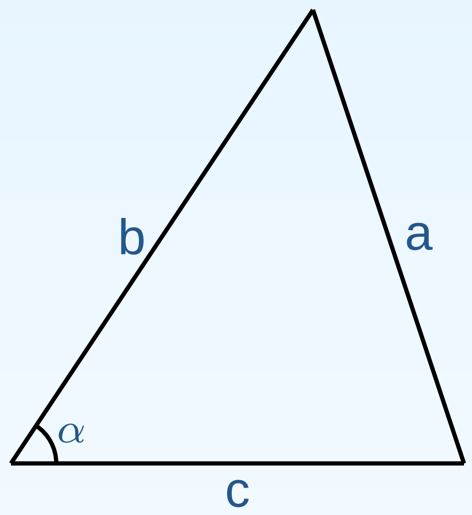
Kosinilause eli laajennettu Pythagoraan lause



$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

Kosinilauseella on kaksi seurausta:

Kosinilause eli laajennettu Pythagoraan lause

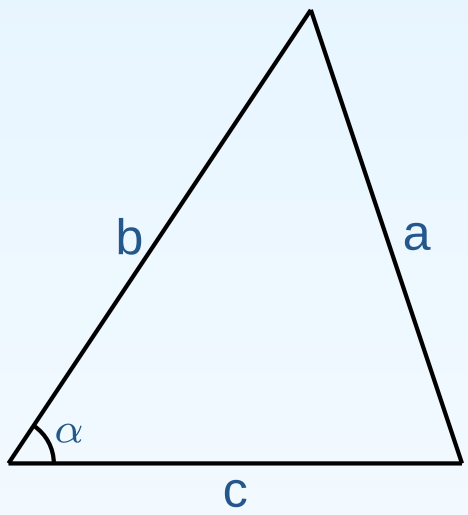


$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

Kosinilauseella on kaksi seurausta:

1. Jos $\alpha = 90°$, niin $\cos\alpha = \cos 90° = 0$, joten kosinilauseen seurauksena saadaan Pythagoraan lause.

Kosinilause eli laajennettu Pythagoraan lause



$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

Kosinilauseella on kaksi seurausta:

1. Jos $\alpha = 90°$, niin $\cos\alpha = \cos 90° = 0$, joten kosinilauseen seurauksena saadaan Pythagoraan lause.
2. **Pythagoraan lauseen käänteislause.** Jos kolmion kahden sivun neliöiden summa on kolmannen sivun neliö, niin kolmio on suorakulmainen.